VIII International Conference on Computational Methods for Coupled Problems in Science and Engineering
COUPLED PROBLEMS 2019
E. Oñate, M. Papadrakakis and B. Schrefler (Eds)

APPLICATION OF FRONTAL SOLUTION METHOD IN DAM-RESERVOIR PROBLEM

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Key words: Dam and Reservior Interaction, Frontal Solution Method, Time Domain Analysis.

Abstract. Solution of the total equations of the phenomenon in time domain is the main target in the current study. The advantages of the method are high accuracy comparing with iterative methods, and possibility of handling the nonlinear affects. Deficiencies are due to high system requirement, the use of same formulation either Lagrangian or Eulerian which are suitable for the structure or reservoir respectively, and solving the variables with very different orders. The proposed method is to take advantage of frontal solution scheme. In this solution method some of the variables are gathered in a front, and solution is performed each time for successive fronts. The size of the front is different for each problem, but it is much less than total variables. Therefore, computational requirement is limited without losing accuracy. The hydrodynamic pressure variables are chosen to be located in front. Therefore some notable changes had to be made in formal scheme. In common version the choosing the variables and placing them in the front is done automatically. While in proposed application the hydrodynamic pressure variables are forced to be located in front. Other advantages are also achieved. Having the hydrodynamic pressure variables, the structure and reservoir are solved separately using their suitable formulations.

This paper studies two methods for analysis of the dam-reservoir system. The first method is based on using frontal method, while the other utilizes iterative schemes. A typical dam-reservoir system is performed by five different meshes. These analyses are compared from efficiency point of view. Frontal method requires less system requirement than iterative methods.

CPU time are calculated for two approaches. All the execution times are calculated on the same hardware to achieve an appropriate criterion for comparison purposes. Compared to iterative scheme, CPU time is decrease in frontal method. As the number of the elements are

increase, execution time is saved more in frontal method even up to 90 percent.

1 INTRODUCTION

The safety of the dams are really important for the people who lived downstream. The response of the concrete gravity dam depends on the lot of factors. For example the interaction of the dam with reservoir and foundation and even the behavior of the concrete have great influence on the response of the dam. The calculation of the hydrodynamic force which acts on the interface of dams as a result of earthquake is important for analysis of the dams.

Several approaches have been proposed to consider this effect. Most of the approaches are involved with an iterative schemes in time domain. Fellippa and Park [1] descripted staggered solution for coupled problems in detail. Staggered approach make us capable to reduce couple problem to subsystems therefor we only deal with symmetric equations for each subsystems [2]. However the stability of the method is conditional and must be care about convergence of the solution of the subsystems at each time step. So, in this approach we need iteration in each time step even in linear problems. And this procedure is time consuming.

Some approaches are direct. The direct method is exact. However this method need massive storage space. The modal method have been used to analyze the dam-reservoir problem [3]. Modal approach is very efficient in time but it can't be used in nonlinear problems.

A lot of work have been done in frequency domain by Chopra and coadjutor on dam-reservoir-foundation interaction [4,5]. The concept of the frequency domain formulation are more difficult than the time domain formulation. And the main disadvantage of the frequency domain was that, it can't be used in nonlinear problems.

The best method for solving the coupled equations, is simulation method. Simulation method in time domain, need massive storage space because of solving the all of equations including structure and fluid including far field variable for each time step. As well as, solving numerically the all variables including displacement and pressure which have different orders is really difficult.

Frontal method enable us to overcome these disadvantages [6]. The principal aim of this paper is to employ frontal solution for solving the dam-reservoir system [7,8]. The frontal method for solving the equations was presented by Irons for the first time[7].

By using frontal solution, all equations are assembled but only a selection of the degrees of freedom are solved. In the other words, by considering the interface pressure degree freedoms in the front, total model including the displacement of the structure and the pressure of the fluid are assembled by frontal method. Then only the interface degree freedoms front, which is pressure acting on the interface, are solved. After finding the values of the pressure which act on the interface of the dam, the structure can be analyzed due to sesmic force and dynamic pressure, and the fluid can be analyzed due to determined boundary conditions, separately. This schema have a lot of advantages: only the value of the pressure which acts on interface compute first and this value is the same as the value of which solved the entire equations. So there is no approximation. Also this schema can be used in nonlinear problems because none of the fields was eliminated or partitioned.

2 METHOD OF ANALYSIS

Dam's equation of motion can be written as below:

$$[M]\ \ddot{u}\ +[C]\ \dot{u}\ +[K]\ u\ =-[M]\ \ddot{u}_g\ -[Q]\ p \tag{1}$$

Where $[M]$, $[C]$ and $[K]$ are mass, damping and stiffness matrices of the dam structure body respectively. $\ddot{u}_g$ is the ground acceleration. u , $\dot{u}$ and $\ddot{u}$ are vectors of displacement, velocity and acceleration of structure body respectively.

The equation of motion for reservoir domain due to earthquake motion can be written as below:

$$[G]\ \ddot{p}\ +[D]\ \dot{p}\ +[H]\ p\ =\ F\ -\rho_f[Q]^T\ \ \ddot{u}\ +\ \ddot{u}_g \tag{2}$$

Where $[G]$, $[D]$ and $[H]$ are assembled matrix of reservoir domain. F is load vector due to boundary condition of the fluid domain. p is the vector of hydrodynamic pressure, $\dot{p}$ and $\ddot{p}$ are vectors of first and second derivation of the hydrodynamic pressure.

The coupled equation of dam-reservoir can be written as below:

$$\begin{pmatrix} M & 0 \\ \rho_f Q^T & G \end{pmatrix}\begin{Bmatrix} \ddot{u} \\ \ddot{p} \end{Bmatrix}+\begin{pmatrix} C & 0 \\ 0 & D \end{pmatrix}\begin{Bmatrix} \dot{u} \\ \dot{p} \end{Bmatrix}+\begin{pmatrix} K & -Q \\ 0 & H \end{pmatrix}\begin{Bmatrix} u \\ p \end{Bmatrix}=\begin{Bmatrix} -M\ddot{u}_g \\ -\rho_f Q^T\ddot{u}_g \end{Bmatrix} \tag{3}$$

Where $[Q]$ is coupling matrix defined as below:

$$[Q]=\int N_s n {N_f}^T ds \tag{4}$$

where N_s and N_f are shape function of structure and fluid respectively.

By using step by step integration scheme of Newmark, the coupled equation of dam-reservoir can be written as below [6]:

$$\begin{pmatrix} \frac{4}{\Delta t^2}M+\frac{2}{\Delta t}C+K & -Q \\ -Q^T & -\frac{1}{\rho_f}G-\frac{\Delta t}{2\rho_f}D-\frac{\Delta t^2}{4\rho_f}H \end{pmatrix}\begin{bmatrix} u^{n+1} \\ p^{n+1} \end{bmatrix}=\begin{bmatrix} F_s \\ F_f \end{bmatrix} \tag{5}$$

where:

$$\begin{aligned} F_s &= -M\ddot{u}_g + M\left(\frac{4}{\Delta t^2}u^n+\frac{4}{\Delta t}\dot{u}^n+\ddot{u}^n\right)+C\left(\frac{2}{\Delta t}u^n+\dot{u}^n\right) \\ F_f &= \frac{\Delta t^2}{4}Q^T\ddot{u}_g - Q^T\left(u^n+\Delta t\dot{u}^n+\frac{\Delta t^2}{4}\ddot{u}^n\right) \\ &\quad - G\left(\frac{1}{\rho_f}p^n+\frac{\Delta t}{\rho_f}\dot{p}^n+\frac{\Delta t^2}{4\rho_f}\ddot{p}^n\right)+D\left(\frac{\Delta t}{2\rho_f}p^n+\frac{\Delta t^2}{4\rho_f}\dot{p}^n\right) \end{aligned} \tag{6}$$

The interaction equations of dam and reservoir may be divided to liquid degrees of freedom, solid degrees of freedom and the interface degrees of freedom. It was shown that in frontal solution, the pressure variables on interface can be kept in front and can be solved without continuing the solution for the all equations. Then, the achieved results can be used as the boundary condition for solving the reservoir and as loading profile for solving the dam. In this way no iteration is needed, there is no loss of accuracy and each media can be solved by using its suitable formulation. The solution procedure is illustrated in **Figure 1**.

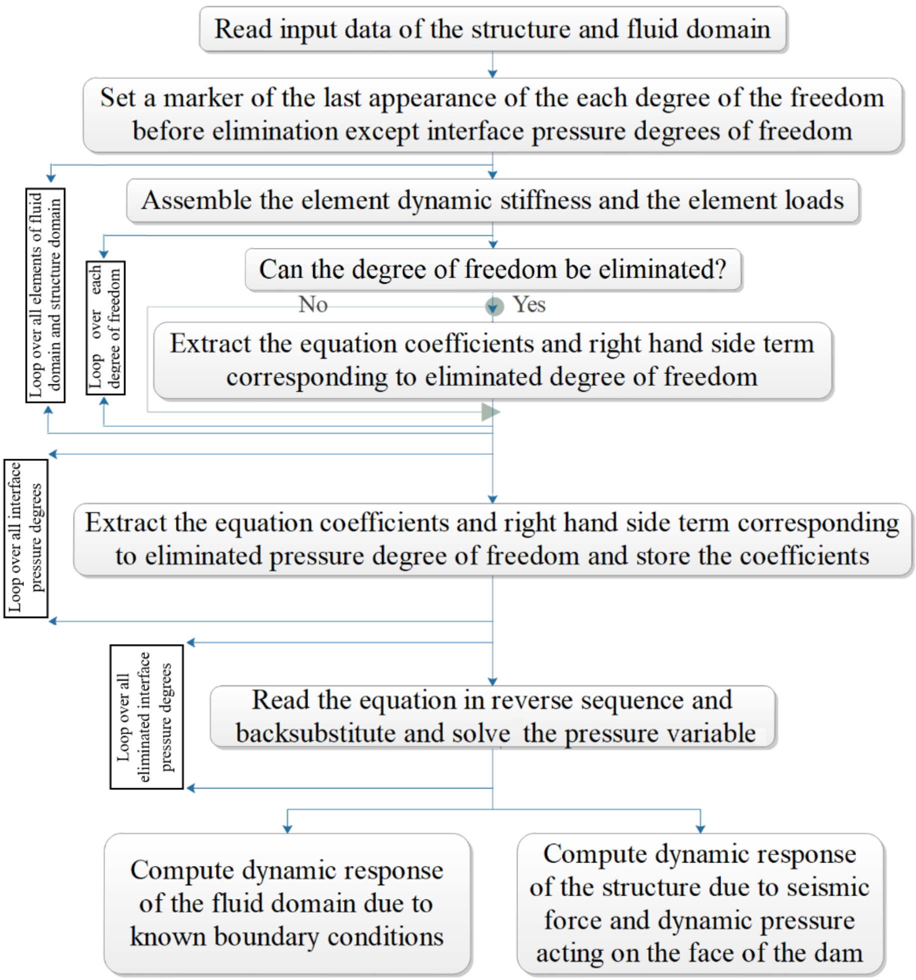

Figure 1: Frontal solution procedure for dam-reservoir problem

3 RESULTS

A computer program is developed based on the frontal solution method. Utilizing this solution, dynamic analysis of Dam with vertical upstream (**Figure 2**) is considered as a typical numerical example. Five types of mesh are used for analysis (**Table 1**).

Table 1: Five types of mesh

	N_s	N_f	N_h
mesh No.1	1	16	10
mesh No.2	2	18	10
mesh No.3	4	36	20
mesh No.4	8	60	36
mesh No.5	8	70	45

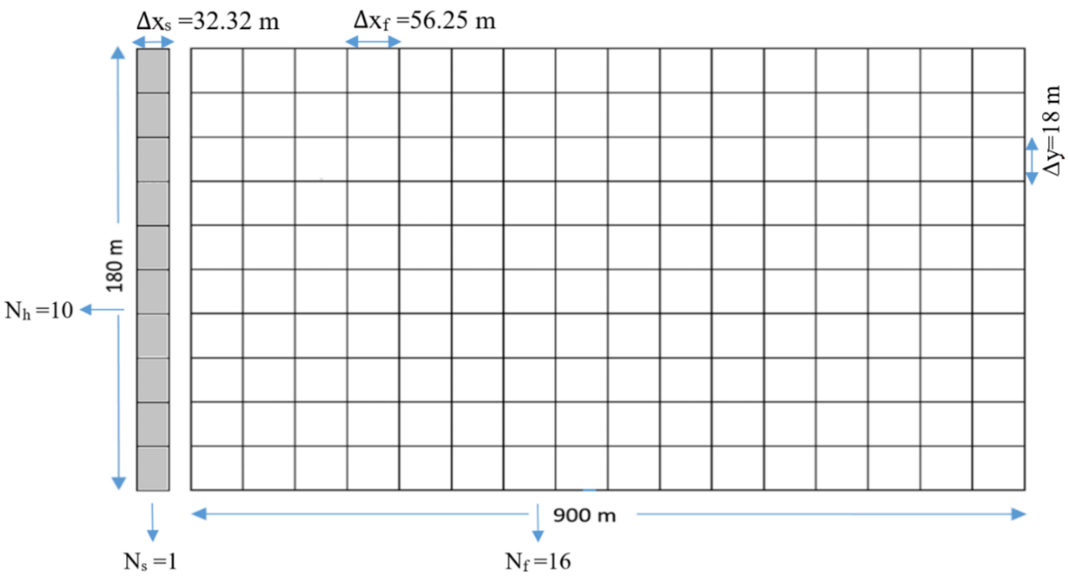

Figure 2: Geometrical detail of Dam-reservoir system (mesh No.1)

A vertical dam with 180m height and reservoir have been subjected to ramp acceleration (**Figure 3**). For the concrete, unit weight and poisson's ratio and modulus of elasticity were taken as 1113.86g/m^3, 0.2 and 35GPa, respectively. For the water, unit weight and sound speed in the water were taken as 1000kg/m^3 and 1440m/s, respectively.

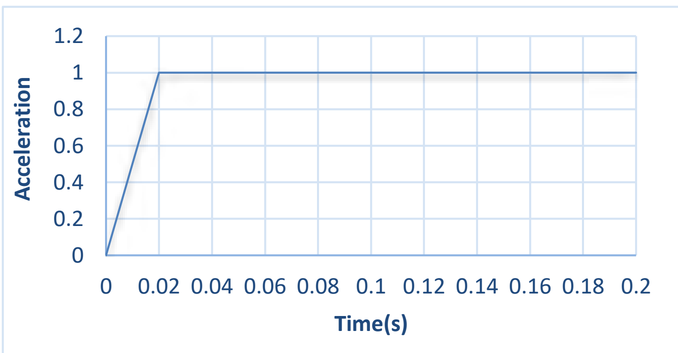

Figure 3: Ramp acceleration

The history of horizontal crest displacement and hydrodynamic pressure at bottom of the reservoir are presented in **Figures 4** and **Figures 5.**

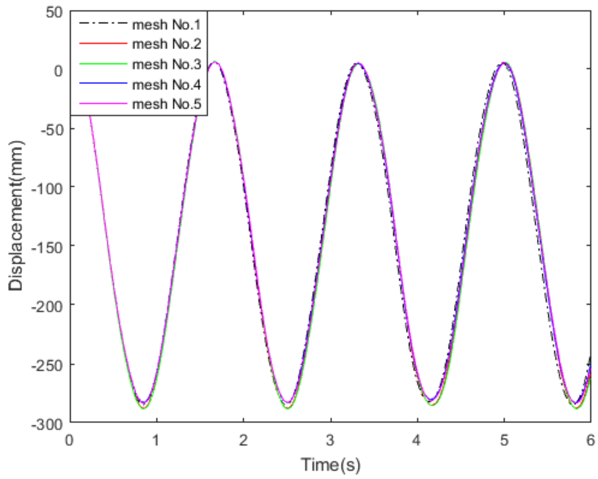

Figure 4: Crest displacement

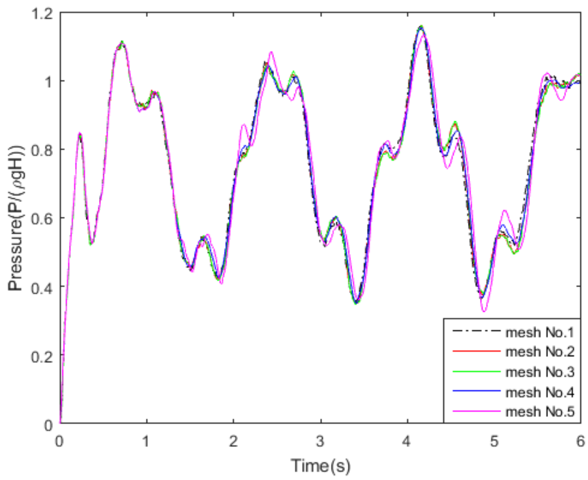

Figure 5: Hydrodynamic pressure at bottom of the reservoir

For comparison purpose, the models were analyzed by using two different program, first program solved the equations by frontal method and second program solved the equations by iteration scheme. The results for mesh No.4 are presented in **Figures 6** and **Figures 7**.

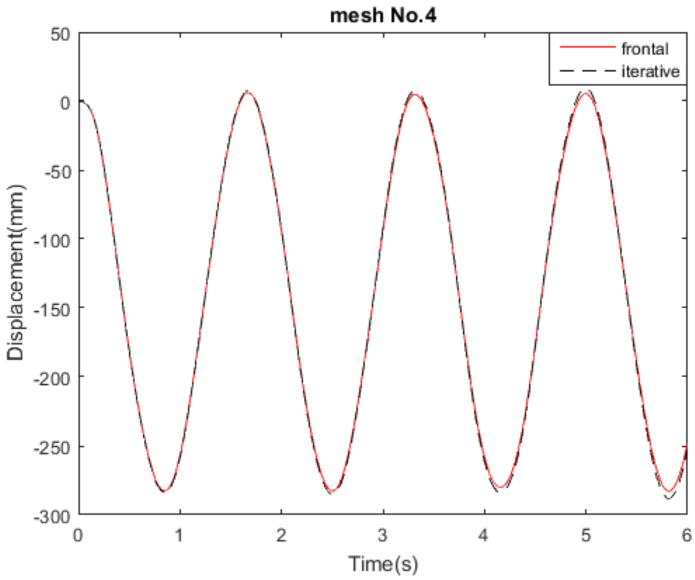

Figure 6: Crest displacement

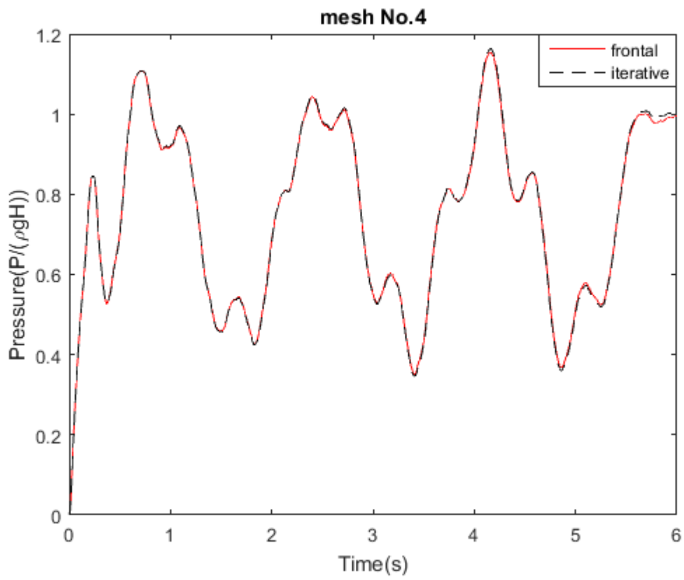

Figure 7: Hydrodynamic pressure at bottom of the reservoir

Table 2 presents the calculation time of two solution scheme for five examples. As shows execution time is saved more in frontal solution even up to 90 percent.

Table 2: Execution time for each analysis (s)

	Frontal method	Iterative method
mesh No.1	20.00	27.36
mesh No.2	23.39	34.98
mesh No.3	217.42	2158.77
mesh No.4	2028.52	45504.80
mesh No.5	3699.98	*

4 CONCLUSIONS

- The frontal solution takes less calculation time than iterative method.
- As the number of the elements are increase, execution time is saved more in frontal method even up to 90 percent.
- The presented scheme is capable of considering the nonlinear phenomenon.
- The presented scheme does not has the geometrical and boundary condition limitation.

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